SAT Math. The essential moves.

QUICK REFERENCE 1 OF 2 · Algebra, functions, rates and data

Start with the named concept, then check its conditions. Pages 3–10 show how these rules solve actual questions from Pursu's bank.

Lines & systems · p3

$y = mx + b \cdot m = \frac{\Delta y}{\Delta x}$

$y - y_1 = m(x - x_1)$

Nonvertical parallel lines: equal slopes. Perpendicular: $m_1 m_2 = -1$ for nonvertical lines. A system's solution satisfies both equations.

Quadratics · p4

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_v = -\frac{b}{2a} \cdot D = b^2 - 4ac$

For $a \neq 0$: $D > 0$ gives 2 real roots; $D = 0$, 1; $D < 0$, none. Vertex form: $a(x-h)^2 + k$.

Structure & domains · p5

$u^2 - v^2 = (u - v)(u + v)$

$a^m a^n = a^{m+n} \cdot (a^m)^n = a^{mn}$

Cancel factors, not terms. Denominators cannot be 0. Even-root radicands must be nonnegative. Check roots after squaring.

Functions & exponentials · p5–6

$g(x) = Af(B(x - h)) + k$

$Q(t) = Q_0 b^{t/T}$

Shift right h, up k; scale vertically by $|A|$, horizontally by $1/|B|$. b is the multiplier per T time units.

Percent & rates · p6

$\%\ \text{change} = \frac{\text{new} - \text{old}}{\text{old}} \cdot 100$

$d = rt \cdot \rho = \frac{m}{V}$

Increase $p\%$: multiply by $1 + p/100$. Decrease: $1 - p/100$. Reverse a change by dividing by the multiplier.

Center & spread · p7

$\bar{x} = \frac{\sum x}{n} \cdot \text{total} = n\bar{x}$

$\bar{x}_{\text{combined}} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

Median: middle after sorting. Range: max − min. IQR: $Q_3 - Q_1$. Adding a constant preserves spread.

Probability · p8

$P(A \mid B) = \frac{\#(A \cap B)}{\#B}$

$P(\text{not } A) = 1 - P(A)$

"Given B" changes the denominator to B only. For an "or," add the groups and subtract their overlap.

Inference · p8

$\text{interval} = \text{estimate} \pm \text{margin of error}$

Random sampling supports generalization to the sampled population. Random assignment supports cause-and-effect conclusions in a well-designed experiment.

Fast problem routing

Constant difference → linear (p3). Constant ratio → exponential (p5–6). Maximum/minimum → vertex (p4). "Given" → conditional denominator (p8). Similar shapes → length / area / volume powers (p9).

Geometry at a glance.

QUICK REFERENCE 2 OF 2 · Given on the test vs. rules to know

GIVEN Included in College Board's reference information. Other rules below are study essentials.

Area & circumference

GIVEN

$A_{\text{rectangle}} = lw \cdot A_{\text{triangle}} = \frac{1}{2}bh$

$A_{\text{circle}} = \pi r^2 \cdot C = 2\pi r$

Height is perpendicular to the chosen base. Use the radius, not the diameter.

Volumes

GIVEN

$V_{\text{box}} = lwh \cdot V_{\text{cylinder}} = \pi r^2 h$

$V_{\text{cone}} = \frac{1}{3}\pi r^2 h \cdot V_{\text{sphere}} = \frac{4}{3}\pi r^3$

$V_{\text{rectangular pyramid}} = \frac{1}{3}lwh$

For cones and pyramids, h is perpendicular height, not slant height.

Right triangles & angle totals

GIVEN

$a^2 + b^2 = c^2$

$45^\circ : 45^\circ : 90^\circ \to x, x, x\sqrt{2}$

$30^\circ : 60^\circ : 90^\circ \to x, x\sqrt{3}, 2x$

Triangle angles sum to 180°. A full circle is $360^\circ = 2\pi$ radians. c is the hypotenuse.

Similarity & surface area · p9

$\text{length} : k \quad \text{area} : k^2 \quad \text{volume} : k^3$

$SA_{\text{box}} = 2(lw + lh + wh)$

$SA_{\text{cylinder}} = 2\pi r^2 + 2\pi rh$

Surface area counts exposed faces. Joining two solids hides both copies of the shared face.

Triangle ratios · p10

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \cdot \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

For acute θ, $\sin\theta = \cos(90^\circ - \theta)$. Match the named angle before choosing sides.

Circles & coordinates · p10

$(x-h)^2 + (y-k)^2 = r^2$

$s = r\theta \cdot A_{\text{sector}} = \frac{1}{2}r^2\theta$

The last two formulas require radians. With degrees, use $\theta/360$ times circumference or circle area. A tangent is perpendicular to the radius.

Coordinates you should know

Distance: $\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$. Midpoint: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$. On the unit circle, a point at angle θ is $(\cos\theta, \sin\theta)$.

Research & question sources

Scope checked against College Board's Math Specifications and Digital SAT Assessment Framework; GIVEN labels checked against Practice Test 10, reference information. Reviewed September 20, 2026. Worked questions: Pursu's internal bank; mathematical typesetting normalized and answers independently checked. Other small examples illustrate the rules.

Linear equations, systems & inequalities.

ALGEBRA · Recognize a constant rate, a shared solution, or a boundary

Build the model

$y = mx + b \cdot y - y_1 = m(x - x_1)$

m: change in y per 1 unit of x. b: value at x = 0, if meaningful in context. Direct variation $y = kx$ has intercept 0.

Solve & classify one equation

$ax + b = cx + d \Rightarrow (a - c)x = d - b$

If $a \neq c$, one solution. If $a = c$, compare constants: equal → all real x; unequal → no solution.

Count system solutions

$ax + by = c,\ dx + ey = f$

For two genuine lines: if $ae - bd \neq 0$, one solution. Proportional left sides with incompatible constants → none. Entire equations proportional → infinitely many.

Graph the boundary

$y > mx + b$ or $y \geq mx + b$

Shade above; use a dashed line for > and a solid line for ≥. For < or ≤, shade below. A system uses the overlap.

Reverse the inequality

$-3x < 12 \Rightarrow x > -4$

Multiplying or dividing by a negative flips the sign. Do not divide by an unknown-sign expression without considering its sign.

Read the requested quantity

$x + y,\ 3x,\ x - y$

The intersection gives x and y. The question may ask for a combination, cost, or difference instead. Horizontal and vertical lines are perpendicular.

PURSU BANK QUESTION Proportional equations

$$\frac{5}{9}x + \frac{8}{9}y = \frac{5}{3}$$
$$px + qy = 15$$

In the given system of equations, p and q are constants. The system has infinitely many solutions. What is the value of $\frac{p}{q}$?

A $\frac{3}{5}$ **B** $\frac{5}{8}$ **C** $\frac{8}{5}$ **D** $\frac{9}{5}$

Recognize → apply → finish

Multiply the first equation by 9: $5x + 8y = 15$. For the second equation to describe the same line, $p = 5$, $q = 8$. Therefore $p/q = 5/8$: **B**.

Trap: Matching only one coefficient is not enough; the constants must scale by the same factor.

DESMOS CHECK Graph both equations to check their relationship. Overlapping lines suggest infinitely many solutions; prove it by comparing the full equations.

Quadratics: choose the useful form.

ADVANCED MATH · Roots, extrema and the number of solutions

Standard form

$$f(x) = ax^2 + bx + c,\ a \neq 0$$

The y-intercept is c. Axis: $x = -b/(2a)$. Vertex: $(x_v, f(x_v))$. Opening up gives a minimum; opening down gives a maximum.

Factored & vertex forms

$$f(x) = a(x - r_1)(x - r_2)$$
$$f(x) = a(x - h)^2 + k$$

Factored form exposes zeros. Vertex form exposes (h, k). With two roots, the axis lies halfway between them.

Exact real-root count

$$D = b^2 - 4ac$$

D > 0: two distinct real roots. D = 0: one repeated real root. D < 0: none. First check that the x^2 coefficient is nonzero.

Root sum & product

$$r_1 + r_2 = -b/a \cdot r_1 r_2 = c/a$$

Use when the question asks for a sum or product rather than individual roots. A root of multiplicity 2 counts twice in these identities.

Complete the square

$$x^2 + bx = (x + b/2)^2 - (b/2)^2$$

Factor out a leading coefficient before completing the square. This converts standard form to vertex form.

Intersect with another graph

$$f(x) = g(x) \Rightarrow f(x) - g(x) = 0$$

For a line and a genuine parabola, one intersection means a repeated root. Apply the discriminant to the difference equation.

PURSU BANK QUESTION

Tangency & a parameter

$$y + m = x + 14$$
$$y - m = x^2 - 8x$$

In the given system of equations, m is a constant. The system has exactly one distinct real solution. What is the value of m?

A $\frac{113}{8}$

B $\frac{123}{8}$

C $\frac{137}{8}$

D $\frac{35}{2}$

Recognize → apply → finish

Eliminate y to obtain $x^2 - 9x + 2m - 14 = 0$. Its discriminant is $81 - 4(2m - 14) = 137 - 8m$. One distinct root requires $137 - 8m = 0$, so $m = 137/8$: **C**.

Trap: A picture that looks tangent is approximate. The discriminant gives the exact parameter.

DESMOS CHECK Graph the original system at $m = 137/8$. It touches at $(9/2, 11/8)$. A slider helps you explore, but does not establish an exact value.

Expressions, domains & functions.

ADVANCED MATH · Preserve equivalence before trusting a solution

Powers & radicals

$$a^m/a^n = a^{m-n} \cdot a^{-n} = 1/a^n$$
$$a^{m/n} = \sqrt[n]{a^m}$$

For positive a, rational exponent rules apply directly. Quotients require nonzero denominators. $\sqrt{x^2} = |x|$, not always x.

Factor first

$$(u \pm v)^2 = u^2 \pm 2uv + v^2$$
$$u^2 - v^2 = (u-v)(u+v)$$

A zero product means at least one factor is zero. If $P(r) = 0$, then $x - r$ is a factor of polynomial P. The remainder on division by $x - r$ is $P(r)$.

Rational & radical restrictions

$$\frac{x^2-9}{x-3} = x + 3 \quad (x \neq 3)$$

Canceled denominators still exclude their original zeros. Even roots need a nonnegative radicand; a principal square root is nonnegative. Squaring can introduce extraneous solutions.

Absolute value

$$|u| = k \Rightarrow u = k \text{ or } u = -k \quad (k \geq 0)$$

For k > 0: $|u| < k$ means $-k < u < k$; $|u| > k$ means $u < -k$ or $u > k$. Negative k gives no solution to $|u| = k$.

Translate & combine functions

$$g(x) = Af(B(x-h)) + k$$

For nonzero A,B: scales are $|A|$ vertically, $1/|B|$ horizontally; negative factors reflect. For $f(g(x))$, evaluate g first. Horizontal shifts act inside f.

Linear vs. exponential

$$f(x) = mx + b \quad \text{vs.} \quad g(x) = Ab^x$$

Equal x-steps give constant differences for a line, constant ratios for an exponential. A = g(0); b > 1 grows, 0 < b < 1 decays (A > 0).

PURSU BANK QUESTION — Rational equations & excluded values

Solve for y:

$$\frac{y^2}{y^2-16} - \frac{2y}{y+4} = \frac{2}{y-4}$$

A 2 **B** 4 **C** 6 **D** 8

Recognize → apply → finish

Exclude $y = 4$ and $y = -4$. Multiply by $(y-4)(y+4)$: $y^2 - 2y(y-4) = 2(y+4)$. Rearranging gives $(y-2)(y-4) = 0$. Keep $y = 2$: **A**.

Trap: The factor y − 4 appears after clearing denominators, but y = 4 was never allowed in the original equation.

DESMOS CHECK Graph the two original sides and inspect their intersections. Keep the domain restrictions even if an algebraically simplified graph fills in a hole.

Ratios, percentages & changing units.

PROBLEM-SOLVING + FUNCTIONS · Identify the base before you calculate

Ratios & proportions

$$a : b = m : n \Rightarrow a = km,\ b = kn$$

If total T is split in ratio m:n, the first part is $Tm/(m+n)$. “a is p% of b” means $a = (p/100)b$.

Greater than vs. of

$$150\% \text{ of } q = 1.5q$$

$$150\% \text{ greater than } q = 2.5q$$

“p% less than” uses $1 - p/100$. To express x as a percent of y, calculate $100x/y$, not $100y/x$.

Successive & reverse changes

$$Q_{\text{final}} = Q_0(1 + r_1)(1 + r_2)$$

Use signed decimal rates. +20% then −20% gives 1.2 × 0.8 = 0.96, a 4% decrease. Divide final value by the product to recover the original.

Exponential time units

$$Q(t) = Q_0 b^{t/T}$$

b is the multiplier every T units. Over Δt units use $b^{\Delta t/T}$. Doubling time T uses b = 2; half-life T uses b = 1/2.

Unit conversion & density

$$d = rt \cdot \rho = m/V$$

Multiply conversion factors so units cancel. 1 ft = 12 in, but 1 ft² = 144 in² and 1 ft³ = 1,728 in³. Square or cube the conversion factor.

Weighted rates & averages

$$v_{\text{avg}} = \frac{\text{total distance}}{\text{total time}}$$

An ordinary mean of two speeds works for equal times, not generally for equal distances. For mixtures, add component amounts before dividing by the total.

PURSU BANK QUESTION — Nested percentages & the correct base

The positive number x is $1,800\%$ of the sum of the positive numbers y and z, and y is 120% of z. What percent of y is x?

A 3100% **B** 3300% **C** 3400% **D** 3500%

Recognize → apply → finish

Let $z = 5$, so $y = 6$. Then $x = 18(y + z) = 18(11) = 198$. Relative to y, $100x/y = 100(198/6) = 3300\%$: **B**.

Trap: 1,800% is a multiplier of 18. The last question changes the comparison base from y + z to y.

DESMOS CHECK Keep grouping visible: type $100 \cdot 18(1.2 + 1)/1.2$. For growth models, evaluate an output ratio to isolate the multiplier.

Data: center, spread & fitted models.

PROBLEM-SOLVING AND DATA ANALYSIS · Ask what the statistic measures

Means use totals

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

For a frequency table, multiply each value by its frequency. Combining groups requires weighting by group size. A removed value equals old total minus new total.

Median, range & quartiles

$$\text{range} = \max - \min \cdot IQR = Q_3 - Q_1$$

Sort before finding the median; with an even count, average the two middle values. Box plots show median and quartiles, not the mean. Histograms show bin counts, not necessarily exact values.

Standard deviation

$$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}}$$

SD measures distance from the mean. Larger distances give larger spread. On SAT comparison questions, reason from the distribution before calculating. This is population SD.

Transform every data value

$$Y = aX + b$$

Mean becomes $a\bar{x} + b$; median transforms the same way. SD, range and IQR multiply by $|a|$. Adding b alone leaves these spread measures unchanged.

Scatterplots & residuals

$$\text{residual} = y - \hat{y}$$

A point above the fitted line has a positive residual. A slope is predicted y-change per x-unit. Predictions far beyond observed data are extrapolations.

Association is not causation

A strong trend does not establish a cause. Check the study design (p8). In a model $\hat{y} = mx + b$, the intercept may lack real-world meaning if x = 0 is outside the context.

PURSU BANK QUESTION — Recovering a removed observation

An engineering firm has 15 designers. The mean number of years these designers have worked with the company is 5.8 years. After one of the designers retires, the mean number of years for the remaining designers drops to 5.2 years. How many years did the retiring designer work at the company?

A 13.6 **B** 14.2 **C** 14.7 **D** 15.3

Recognize → apply → finish

Before retirement, total years = $15(5.8) = 87$. After retirement, total years = $14(5.2) = 72.8$. The retiring designer worked $87 - 72.8 = 14.2$ years: **B**.

Trap: Subtracting the two means gives 0.6, not the missing observation. The group sizes changed.

DESMOS CHECK For a list L, use mean(L), median(L), and stdevp(L) for population SD. For paired data, enter a table and fit $y_1 \sim mx_1 + b$; interpret m in context.

Probability, samples & statistical claims.

PROBLEM-SOLVING AND DATA ANALYSIS · Restrict the group; restrict the claim

Conditional probability

$P(A \mid B) = P(A \cap B)/P(B)$

For a uniformly chosen item, count both A and B in the numerator and B alone in the denominator. Requires P(B) > 0.

Or, and, at least one

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

For independent events, $P(A \cap B) = P(A)P(B)$. “At least one” can be 1 − P(none). Without replacement, later probabilities usually change.

Estimate a population count

$\hat{p} = k/n \cdot \text{estimated count} = N\hat{p}$

Use a representative random sample. The estimate applies to the population sampled, not automatically to a larger or different population.

Margin of error

$[\hat{p} - E, \hat{p} + E]$

Estimate 45%, margin 4 percentage points → 41% to 49%. It estimates a population parameter, not individual outcomes. Larger random samples generally reduce the margin; higher confidence generally increases it, all else equal.

Random selection vs. assignment

Random selection supports generalization. Random assignment to treatments supports causation in a well-designed experiment. One does not substitute for the other.

Bias is not fixed by size

Convenience samples and voluntary polls may be biased. More responses do not repair a biased selection method. An observational association alone cannot prove causation.

PURSU BANK QUESTION — Conditional denominator & grouped counts

There are 4 rows of apple trees and 3 rows of cherry trees in an orchard. Every row of apple trees contains 7 trees taller than 15 feet and 5 trees shorter than 15 feet. Every row of cherry trees contains 6 trees taller than 15 feet and 4 trees shorter than 15 feet. Suppose a tree from these rows is chosen at random. What is the probability that the tree is a cherry tree, given that it is taller than 15 feet?

A $\frac{7}{23}$ **B** $\frac{9}{23}$ **C** $\frac{10}{23}$ **D** $\frac{13}{23}$

Recognize → apply → finish

Tall apple trees: $4(7) = 28$. Tall cherry trees: $3(6) = 18$. Restrict to the 46 tall trees: $P(\text{cherry} \mid \text{tall}) = 18/46 = 9/23$: **B**.

Trap: The denominator is not all 78 trees, because “given taller than 15 feet” restricts the sample space.

	Tall	Short
Apple	28	20
Cherry	18	12
Total	46	32

Restrict to the Tall column.

Geometry: dimensions, scale & hidden faces.

GEOMETRY · Label the lengths before using a formula

Parallel lines & angles

$$\text{straight angle} = 180^\circ$$

Vertical angles are equal. With parallel lines, corresponding and alternate interior angles are equal; same-side interior angles add to 180°. An exterior triangle angle equals the two remote interior angles.

Triangles & similarity

$$\text{length} \to k \quad \text{area} \to k^2 \quad \text{volume} \to k^3$$

AA proves similarity; corresponding side ratios match. Similarity need not imply congruence. For triangle side lengths a,b,c: $|a-b| < c < a+b$.

Area choices

$$A_{\text{parallelogram}} = bh$$
$$A_{\text{trapezoid}} = \frac{1}{2}(b_1+b_2)h$$

For any triangle, A = bh/2 using perpendicular height. For an equilateral triangle, $A = \sqrt{3}\,s^2/4$. Split composite figures into familiar regions.

Prisms, cylinders & pyramids

$$V_{\text{prism}} = Bh \cdot V_{\text{pyramid}} = \frac{1}{3}Bh$$

B means base area. Similar solids use cubed length ratios for volume. If given an area ratio, take its square root before cubing.

Exposed surface area

$$SA_{\text{joined}} = SA_1 + SA_2 - 2A_{\text{shared}}$$

Use this when joining solid faces with no overlap of interiors. Open containers omit missing faces. Do not count a shared face as exposed.

Scale changes compound

$$V_{\text{cylinder}} = \pi r^2 h$$

Doubling r and tripling h multiplies volume by $2^2 \cdot 3 = 12$. Doubling every length multiplies surface area by 4 and volume by 8.

PURSU BANK QUESTION Composite solids & hidden faces

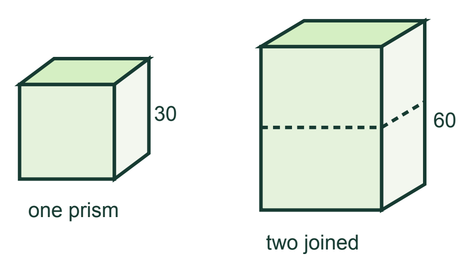
Two identical right rectangular prisms each have a height of 30 centimeters. The base of each prism is a square, and the surface area of each prism is $S\ \text{cm}^2$. If the prisms are glued together along a square base, the resulting prism has a surface area of $\frac{7}{4}S\ \text{cm}^2$. What is the side length, in centimeters, of each square base?

A 10 **B** 12 **C** 15 **D** 20

Recognize → apply → finish

Let s be the base side. One prism: $S = 2s^2 + 120s$. Joined: $2s^2 + 240s$. Set $2s^2 + 240s = \frac{7}{4}(2s^2 + 120s)$. Then $\frac{3}{2}s(s-20) = 0$; since s > 0, **s = 20, D**.

Trap: The two glued square faces disappear from the outside. Discard the zero-length algebraic solution.



Circles & trigonometry.

GEOMETRY AND TRIGONOMETRY · Match the angle, then match the unit

SOH–CAH–TOA

$$\sin\theta = o/h \cdot \cos\theta = a/h \cdot \tan\theta = o/a$$

In a right triangle, o and a depend on the named acute angle. $\sin^2\theta + \cos^2\theta = 1$. For complementary acute angles, sine and cosine swap; tangents are reciprocals.

Radians & unit-circle signs

$$\theta_{\text{rad}} = \theta_{\text{deg}} \cdot \pi/180$$

Unit-circle point: (cos θ, sin θ). Cosine is the x-coordinate; sine is y. Their signs follow the quadrant. Choose degree or radian mode to match the input.

Arc length & sector area

$$s = r\theta \cdot A = \tfrac{1}{2}r^2\theta \quad (\theta \text{ in radians})$$

In degrees use $(\theta/360)2\pi r$ and $(\theta/360)\pi r^2$. Example: r = 6, θ = π/3 gives arc 2π and sector area 6π.

Circle equations

$$(x-h)^2 + (y-k)^2 = r^2$$

Center (h,k); radius r, not r². Complete squares to convert general form. $x^2 + y^2 + Dx + Ey + F = 0$ has center $(-D/2, -E/2)$.

Tangents & inscribed angles

A tangent is perpendicular to the radius at contact. An inscribed angle is half the measure of its intercepted arc; a central angle equals that arc's measure. An angle subtending a diameter is 90°.

Circle from expanded form

$$r^2 = (D/2)^2 + (E/2)^2 - F$$

A real circle needs r² > 0. For $x^2 - 6x + y^2 + 10y = -9$, complete squares: $(x-3)^2 + (y+5)^2 = 25$; center (3,−5), radius 5.

PURSU BANK QUESTION

Similar triangles & tangent-to-sine conversion

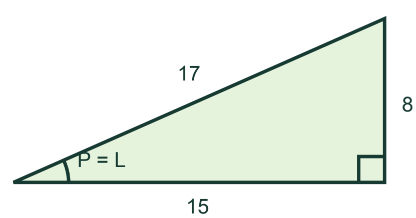
Triangle PQR is similar to triangle LMN, where angle P corresponds to angle L and angles R and N are right angles. The length of PQ is 1.8 times the length of LM. If $\tan(P) = \frac{8}{15}$, what is the value of $\sin(L)$?

A $\frac{8}{17}$ **B** $\frac{9}{17}$ **C** $\frac{10}{17}$ **D** $\frac{11}{17}$

Recognize → apply → finish

Choose legs opposite P = 8 and adjacent P = 15. The hypotenuse is $\sqrt{8^2 + 15^2} = 17$. Since P corresponds to L, $\sin L = \sin P = 8/17$: **A**.

Trap: The 1.8 scale factor changes lengths, but not corresponding angles or trig ratios.



DESMOS CHECK Check with $\sin(\arctan(8/15))$. For circle tangents, compute the radius slope first, then use the perpendicular slope. Vertical radii have horizontal tangents, and vice versa.